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ABSTRACT

This analysis of 44 research studies concerned with word problems contains: (1) a brief overview; (2) 44 major findings and conclusions; (3) 64 suggestions and questions for further research; and (4) a reference list. The author suggests that the experimental research of the 1980s seemed to focus on the children, the instruction, and the word problem itself. It is further suggested that the studies seemed to indirectly indicate that the principles of developmental psychology need to be respected and applied, even when teaching for specific behaviors and outcomes. (PK)

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WORD PROBLEM SOLVING

What is experimental research saying about mathematics word problem solving?

A review of forty-four studies completed between 1980-1987.

Presented at the

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by

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MATHEMATICS WORD PROBLEM SOLVING: A REVIEW OF FORTY-FOUR STUDIES COMPLETED BETWEEN 1980-1988

Respectfully presented at the 66th Annual Conference of the National Council of Teachers of Mathematics, April 6-9, 1988, Chicago, Illinois by Ned V. Schimizzi, Associate Professor of Education, Department of Elementary Education and Reading, State University College at Buffalo, New York 14222

A problem is not necessarily 'solved' just because the correct response has been made. A problem is not truly solved unless the learner understands what he/she has done and knows why his/her actions were appropriate (Brownell, 1942).

How often do teachers accept a correct conditioned response for conceptualization?

INTRODUCTION

In its Agenda for Action: Recommendations for School Mathematics of the 1980's, the National Council of Teachers of Mathematics strongly recommended that problem solving be the major focus of school mathematics. The NCTM recommendations were a result of studies conducted by the National Assessment of Educational Progress and the National Science Foundation. These studies revealed children's extremely poor performance in the problem solving skills.

The number of papers concerned with problem solving which have been published since the NCTM statement was made, abound. These efforts have been sincere responses to NCTM's recommendation and deserve to be read and considered. The studies reviewed in this document, however, related to solving word problems. They were published since 1980 when the NCTM recommendation was made.

The exclusion of experimental studies completed prior to 1980 in this document was because of resource limitations, especially time. Many of these prior studies were landmark contributions to the teaching of word problem solving. They too deserve to be read and considered.

The experimental research of the 1980's seemed to focus on the children, instruction, and the word problem itself. The studies which focused on the children were concerned with these five major topics: (1) the children's own inventive processes for solving word problems; (2) conceptual understanding; (3) cognitive functioning; (4) difficulties with two-step problems; and (5) concept representation by using diagrams and drawings. Although one or two studies analyzed three-dimensional manipulatives for conceptual representation, there seemed to be a shortage of such studies.

The studies which focused on instruction were concerned with these eight major topics: (1) instructional models; (2) explicit instructions and explanations; (3) direct sequenced instruction; (4) small cooperative groups with some peer group instruction; (5) estimation as a method for pupils to preview a solution process; (6) the requirement that children restate the problem in their own words as a method of encouraging correct interpretation; (7) computer assisted instruction for teaching and reinforcing problem solving skills; (8) computer models for use in word problem solving research. There seemed to be a shortage of experimental studies which included the use of computers.

The studies which focused on the problem itself were concerned with: (1) language; (2) readability; (3) structure; (4) format; and (5) syntax.

Experimental studies which focused on rating textbook series for effectiveness in preparing children for solving word problems seemed to be non-existent. Rating scales for measuring this kind of textbook effectiveness seem to be rare.

These studies seemed to indirectly indicate that the principles of developmental psychology need to be respected and applied even when teaching for specific behaviors and outcomes. More studies seem to be needed which might reveal the respective and combined values of developmental and behavioral psychologies for more effective children's word problem solving abilities. The emerging concern seems to be not that children know merely how to achieve a solution to a word problem but that they understand why the solution is correct, i.e., conceptualization should accompany an instructed process.

MAJOR FINDINGS AND CONCLUSIONS LISTED

1. Teachers who provided explicit explanations and interacted meaningfully with students were more effective in helping them become successful problem solvers (Herrmann, 1986).

2. Children at the same conceptual level seem to differ in their ability to construct and maintain representations of sensory motor actions (Cobb, 1986).

3. There are some children who are able to reason about quantitive problems, i.e., they know the basic procedures of a given operation but may not use algorithmic procedures to find answers to verbal

problems. They may rely on direct modeling and counting (Romberg, 1983).

4. Early didactic models in a child's schooled learning experiences might exert considerable influence on problem solving behavior even after the acquisition of formal mathematical thought (Fischbein, 1985).

5. Four skill areas represented an immediate need for a sizeable number of sixth graders, i.e., computation, interpretation, reading, and the integration of the three previous skills for finding a solution for a word problem (Ballew and Hunter, 1983).

6. One possible effect of some kinds of instruction on problem solving behavior might be a shift from a variety of strategies to solve a variety of problems to a single strategy (Carpenter, 1981).

7. Asking the child to restate the problem correctly in his own words can help the teacher identify his/her representations of the problem. It might also help the child better understand the problem (Quintero, 1983).

8. Teachers can be more sensitive to the sequence of instruction when they understand the prerequisite

knowledge structures for solving certain problems. Different strategies can be adopted when teaching at different levels (Nesher, 1982).

9. In regards to word problems, teachers should help students to develop the ability to break down complex-compound sentences into several simple sentences (Wheeler and McNutt, 1983).

10. Students working in cooperative conditions outperformed students in individualistic competitive conditions when solving word problems (Johnson and others, 1980).

11. The use of drawings to organize the data in word problems was most helpful to students scoring low on cognitive ability tests (Threadgill-Sowder and others, 1985).

12. The use of diagrams, the appropriate reordering of number sequences, and the removal of extraneous information can improve the success rate in solving word problems (Cohen, 1981).

13. The problem structure and the overall pattern of relations between the quantities in the problem had an effect on problem solving success (Shalin and others, 1985).

14. Field-independent subjects solved more word problems correctly than did field-dependent subjects (Vaidya and others, 1981).

15. Most of the children had more difficulties with two-step word problems than one-step problems. These difficulties were conceptual and strategic. Children tended to use the same strategies for solving two-step word problems as they did for one-step problems, i.e., with a single strategy. Students improved their performance in the two-step word problems after having been told that the problems required a two-step solution (Quintero, 1984).

16. Instruction had a bearing on any changes in a child's invented behavior for solving word problems. Instruction had a major effect on the range and application of learned strategies (Moser and others, 1982).

17. When working in small groups, a solution to a word problem was available within three acts of the reading of the problem. In each group, one student contributed very little of the total answers and another student gave over half the answers (Guilbert and Leitz, 1982).

18. Presenting word problems by way of drawings was clearly

more effective than the standard words-only presentation (Threadgill-Sowder, 1982).

19. The small group working without teacher intervention seems to be a powerful learning tool (Noddings, 1982).

20. Children trained to solve basic types of word problems in a sequence achieved higher posttest scores than those children who were trained to solve an unsequenced arrangement of the same problems (Jones and others, 1985).

21. Reading ability played a major role in the solution of word problems and extraneous information reduced the accuracy of students' answers and increased the length of their test-taking times (Muth, 1982).

22. The direct instruction approach which included the teaching of prerequisite skills in a sequential manner and explicit teaching of problem-solving skills was more successful than a traditional approach. Time on task alone did not increase performance (Darch and others, 1983).

23. The practice of applying readability formulas to determine the grade level appropriateness of word problems was questioned. Readability scores a few

grades above or below grade level did not substantially affect students' abilities to solve word problems (Paul, 1986).

24. Most first graders were limited to direct symbolic representations of word problems (Carpenter and others, 1985).

25. Sixth graders rarely applied systematic problem analysis when confronted with a word problem. Their difficulties with word problems seemed to be caused by a lack of the attitudes and skills necessary to the act of analyzing a problem before attempting to compute a solution (DeCorte and others, 1981).

26. Problem solving skill might be acquired from sources other than formal schooling. This might be especially true in other cultures (Adetula, 1985).

27. There was a high correlation between the problems solved and those restated by the children correctly in their own words. A major source of the children's difficulty was in repeating the intensive quantity. The representation of the intensive quantity was particularly important in solving these word problems (Quintera, 1981).

28. (a) Children relied less on verbal and material strategies and more on mental strategies as the school year progressed. (b) The value of data collected by human beings observing the behavior of other human beings might surpass that of computer models indefinitely (DeCorte and others, 1985).

29. The concept of ratio and two-step word problems were a source of difficulty for children (Schwartz/Quintero, 1981).

30. More effective use of children's natural ability to solve verbal problems might be possible with properly modified instruction (Moser and Carpenter, 1982).

31. Instruction increased pupils' success in writing number sentences (Moser and Carpenter, 1982).

32. Computers might allow children to represent problems in a formal way without the learners having completely mastered the formal algorithms and number facts (Moser and Carpenter, 1982).

33. Students who generated their own diagrams to represent word problems were more successful in solving them than students who did not (Yancy, 1981).

34. A model instructional unit was an apparent effective means of teaching mathematics story problem solving skills when it was accompanied by group involved discussions, illustrations, and demonstrations of word problem solutions (Wilson, 1982).

35. Teaching children to represent problem situations through the use of various models increased their ability to solve word problems (Rathnell, 1981).

36. Children improved their word problem solving performance when they were instructed to identify the wanted and the given in each problem i.e,, if the parts are given, add to find the whole. If the whole and a part are given, subtract to find the unknown part (Rathnell, 1981).

37. The order of presentation of two given numbers affects children's success when solving word problems. Children are more successful when the smaller number is given first DeCorte and VerSchaffel, 1987).

38. The order of presentation from less difficult to more difficult problems produced positive transfer of learning and more successful problem solving. Problem embodiment or wording was even more influential (Zollman, 1987).

39. Children with learning disabilities can improve their word problem solving performance with computer assisted training procedures (van Lieshout, 1986).
Girls learned the components of the problem text.
Boys learned to draw a diagram representing the problem to be solved.

40. Whether a word problem has a readability score a few grades below, at, or above a given grade level, there is no substantive effect on the students' ability to solve it (Douglas and Nibbelink, 1986).

41. Personalized test items improved performance on the part of the students to choose the correct mathematical process to solve a problem (Wright, Jane and Dan, 1986).

42. The semantic structure of simple addition and subtraction word problems seriously influences children's solution processes (DeCorte and Ver Schaffel), 1986 and 1987). Evidence was supported by eye-movement data.

43. Students who had high spatial visualization skills solved no more mathematics problems than students who had no spatial visualization skills (Fennema and Tarte, 1985).

44. Students who process mathematics information by verbal-logical methods outperformed students who processed mathematical information visually (Lean and Clements, 1981).

SUGGESTIONS AND QUESTIONS FOR FURTHER RESEARCH IN MATHEMATICS WORD PROBLEM SOLVING*

1. A focus on the qualitative dimensions of instruction provided by teacher educators should include studies which: (a) describe characteristics of the verbal interaction patterns of students who actively interpret instructional information; and (b) teacher educators who actively interpret the student responses to instruction.

2. Does the study of the logo computer language enable the student to solve word problems more effectively?

3. Are some step-by-step problem solving strategies more effective than others? For whom? Why?

*Please note that most of these suggestions and questions were the reviewer's own ideas. They were a result of his reading and analyzing each study. Some of the suggestions were explicitly stated by the researchers and an effort was made to identify those items for the reader.

4. Do teachers apply the work of Piaget when explaining and giving directions for word problems, i.e., do teachers include considerations for psychological, developmental readiness as well as mathematical readiness?

5. What can teacher educators do to encourage teachers to apply the work of Piaget when explaining and giving directions for word problems?

6. To what degree do repeated exercises in drill and practice cause mathematical mindsets which inhibit formal thinking and cause children to focus on single strategies for solving word problems?

7. How can we avoid the development of mathematical mind sets? Can we do so by efforts toward helping children develop an understanding of our number system?

8. Can the time at which a child enters the formal operations stage be affected by teaching? If so, how? If not, why not?

9. How much do instruction and instructions contribute toward molding children into convergent thinking patterns and the use of single strategies for solving a variety of problems? Is convergent thinking a

desirable outcome in the teaching of mathematics? How much does our concern for standardized test scores contribute toward the teaching for convergent thinking?

10. How often do teachers mistake a correct conditioned response for an internalized concept (conceptualization)?

11. How often do teachers teach concrete operational children (Piaget) as if they were formal thinkers? or How often do teachers respect both performance and developmental factors when teaching mathematics?

12. Should mathematics teachers also be teaching some language arts and reading? Should some reading and language arts teachers also be teaching some mathematics? How can the efforts of mathematics teachers and teachers of reading and language arts be combined for the purpose of improving student abilities in solving word problems?

13. Should the teaching of mathematics in the elementary school be multi-tracked? If so, should materials for a given grade level also be multi-tracked in difficulty including the syntax of word problems?

14. When should teachers supplement their own methods and strategies for teaching a given concept or skill with children's peer group instruction?

15. Are we moving into an era when conditions and tools for communicating demand that formal thought itself be redefined?

16. Should formal thought be redefined to include the cognitive restructuring of abstract concepts for slow and average learners?

17. Are we misassuming the existence of formal thought in some learners, i.e., in practice, do our teaching methods and materials assume that learners become formal thinkers simultaneously?

18. Should formal thought be redefined or cognitively restructured for unimpeded and gifted learners to include advanced formal thought, i.e., a computer language such as logo which is considered an advanced form of abstract thinking for some children? Indeed, logo is thought by some (Papert, 1980) to have the capacity to alter the problem solving process.

19. For how many days, weeks, months or years shall learners retain the ability to apply specific

mathematical operations for solving word problems when these operations were "behavioralistically" acquired without respect for the principles of developmental psychology?

20. Should teachers and publishers remove and edit out any structural format faults which interfere with student success in solving word problems?

21. Which specific difficulties that children have in solving word problems can be attributed to the way the problems are written?

22. Is there a correlation beween left or right brain hemisphere dominance and field dependence or independence?

23. Some studies indicate that successful mathematics students might be left-brain dominant. If so, are left-brain dominant students field dependent or field-independent?

24. Some studies indicate that field independent students are more successful in mathematics than field-dependent students. Should the development of a field independent cognitive learning style be encouraged in learners? If so, how should it be encouraged?

25. How might instruction (convergent thinking) increase children's effectiveness in solving word problems without inhibiting the development of their inventiveness (divergent thinking)?

26. If indeed, participation in small groups helps children become more effective problem solvers, is it because of the small group dynamics at work? The peer group instruction? Or a combination of both? Do only the students who are active participants in the group experience gains? Can gains be made in the problem solving skills by 'silent' participation in the group? How much does the emerging group 'pecking order' determine each member's success in that group? What are the factors which determine the 'pecking order' within the group, i.e., status and sociability, math and verbal ability, one's gender?

27. What are the long term effects on concept retention of rote, repetition, conditioning methods, and the meaningful applications of computational operations to manipulations of concrete objects and events? Is it possible to combine the best of these two schools of thought (behavioral and developmental psychologies)? If so how?

28. Should the teaching of reading be combined with computational skills as a method for developing

and assessing comprehension while simultaneously teaching and reinforcing problem solving skills? How much longer can the efforts of reading and mathematics teachers remain separated?

29. How much do traditional textbooks, curriculums and teaching strategies contribute toward student success in word problem solving? <u>or</u> How much do they contribute toward distracting from success, i.e., do they provide for the prerequisite sequenced skills which are needed for developing and bringing together the necessary conceptualizations, computational skills, and strategies for word problem solving?

30. The 'new mathematics' relied heavily on the contributions from mathematicians. Should the mathematics for the 1990's and the new century seek and consider the contributions of the developmental psychologists?

31. How much of word-problem solving is an act of creativity?

32. To what degree does investigation itself change the character or process of that which is being investigated (Harris, 1986)?

33. Do the cognitive processes of persons over 50 years

of age differ from those of younger-persons? If so, then why? What effects did the absence of television in the lives of persons over 50 years of age have on their cognitive processes?

34. Erik DeCorte recommended the following hypothesis for further study: The acquisition by the learners of the heuristic estimation strategy leads to a qualitative improvement in their word problem solving, i.e., it is especially the heuristic estimation strategy which induces an improvement in learner's word problem solving and is the determining factor of the increase in that performance. In researching this hypothesis DeCorte recommended that special attention be given to the collection of qualitative data on pupils' problem solving processes before, during, and after the experimental teaching program.

35. Was it the estimation of the outcome itself that made a difference in DeCorte's learner's word problem solving abilities or was it the abbreviated 'passing through' the solution process, i.e., the 'dry-run' which was encouraged by the estimation? What other factors could be applied to encourage the 'dry-run' through a word problem?

36. Simple ratios and simple fractions seem to be highly

abstract concepts for children. Why? Do children have difficulties with the concepts of simple ratios and simple fractions for some of the same reasons they have difficulties with word problems?

37. In the minds of teachers, what is teaching for conceptualization?

38. Do teaching strategies for word problems which combine conceptual development with process result in increased retention as compared with teaching process alone?

39. re: Moser, James M., Carpenter, Thomas P. (1982). How much of the children's improvement in performance could be attributed to the increased conceptualization of the mathematical processes involved and how much of this increased conceptualization could be attributed to the Piagetian-type two-dimensional representations of the word problems appearing on the computer displays?

The following ten questions for further research were contributed by Anna Vance Yancy (1981) in her study titled: Pupil Generated Diagrams as a Strategy for Solving Word Problems in Elementary Mathematics:

40. Would the method (conceptual imagery through drawing) be of more or less benefit to pupils of more or less

aptitude than those subjects in this study?

41. Would the effect be more or less pronounced if compared to truly traditional instruction rather than the Method A/ Method B comparison used in this research?

42. Would the typical teacher adopt the drawing imagery techniques with those positive effects experienced by teachers in this study?

43. Would the technique be more or less effective at other grade levels?

44. Do the acquired pupil skills represent a permanent improvement in word problem solving ability?

45. Would the technique be as effective in other curriculum areas, eg., science?

46. Could the technique be successfully taught to students by workbooks which show pupils how to diagram the inherent structure of word problems?

47. Which type of learner would benefit most from which method of instruction?

48. Could student generated diagrams be used to diagnose

individual pupil difficulties with mathematical concepts?

49. Does the inability to diagram a problem indicate that a student does not understand the concepts involved, even if he calculates a correct answer?

50. How much do mathematics textbooks contribute or distract from children's success in word problem solving?

51. Have any rating scales been developed which recognize the presence or absence of specific strategies in mathematics textbooks which are known to contribute toward the encouragement of children's success in solving word problems?

52. Are curriculum concerns best served by Skinnerian principles?

53. Are children's needs best served by Piagetian concerns?

54. In the best interests of the efforts to improve children's problem solving abilities should the most meaningful contributions of both schools of thought, behavioral and developmental, be combined in order to maximize the effectiveness of these efforts? Should

55. Why doesn't more of the research concerning wording focus on older children in grades five through nine?
56. How might instruction increase children's effectiveness in solving problems without inhibiting the development of their divergent (inventive) mental processes?
57. How important is it that first graders do word problems? second graders?
58. What are the effects of different task characteristics on children's strategy choice(s) in each developmental stage. (DeCorte and VerSchaffel, 1987)
59. To what extent are children aware of the factors that determine their strategy choice and how this relates to their knowledge of mathematical principles such as the commutativity principle and the complementarity of addition and subtraction. (re: DeCorte and VerSchaffel, 1987)

Teachers and researchers who are investigating the process of determining the effects of grouping on children's success in solving word problems should ask these questions: (re: Gilbert and Lietz, (1982)

60. Do only the students who are active participants in the group discussions experience gains? If so, why?
61. Can gains in acquiring the problem solving skills be made by "silent" participation in the group?
62. To what degree does the emerging group pecking order determine one's success in the group?

63. What are the factors which determine the pecking order in the group? i.e., Status and sociability, mathematics and verbal abilities, gender?

64. A possible paradox: Does the entire education effort unintentionally 'stack the deck' against word problem solving then laments the results and pleads for solutions; a victim of its own size and complexity which cannot always readily change itself?

REFERENCES

Adetula, L. O. (1985). The effects of language and schooling on the solution of simple word problems by Nigerian children (Report No. SE 046-313). Madison, Wisconsin: Wisconsin Center for Education Research. (ERIC Document Reproduction Service No. ED 264 143)

Ballew, H. & Cunningham, J. W. (1982). Diagnosing strengths and weaknesses of sixth grade students in solving word problems. Journal for Research in Mathematics Education, 13(3), 202-210.

Briars, D. J. & Larkin, J. A. (1984). An integrated model of skills in solving elementary word problems. Cognition and Instruction, 1(3), 245-296.

Brownell, W. A. (1942). Problem solving, psychology of learning. In 41st Yearbook of the National Society for the Study of Education (Pt. 2, p. 439). Chicago: Society for the Study of Education.

Carpenter, T. P., Hiebert, J. & Moser, J. M. (1983). The effect of instruction on children's solutions of addition and subtraction word problems. Educational Studies in Mathematics, 14, 55-72.

Carpenter, T. P. & Others (1985). The representation of basic addition and subtraction word problems (SE-045-940). Madison, Wisconsin: Wisconsin Center for Educational Research. (ERIC Document Reproduction Service No. ED 260 905)

Cobb, P. (1986). Counting types and word problems. Paper presented at the Annual Meeting of the American Educational Research Association. San Francisco, California. The research reorted in this paper was supported by a grant from the Spencer Foundation.

Cohen, S. A. & Stover, G. (1981). Effects of teaching sixth grade students to modify format variables of mathematics word problems. Reading Research Quarterly, XVI(2), 175-199.

Copeland, R. W. (1979). Children learn mathematics (3rd. ed.). New York: Macmillan Publishers.

Darch, C. & Others (1983). Instructional approaches and levels of practice in working fourth grade word problems (SE-043-179). Montreal, Quebec, Canada: American Educational Research Association. (ERIC Document Reproduction Service No. ED 235 010)

DeCorte, E. & Somers, R. (1981). Estimating the outcome of a task as a heuristic strategy in arithmetic problem solving: A teaching experiment with sixth-graders (Report No. 27, SE-034-929). Leuven, Belgium: Katholieke Universiteit. Paper presented at the Annual Meeting of the American Educational Research Association. Los Angeles, CA: April 13-17, 1981. (ERIC Document Reproduction Service No. ED 202 693)

DeCorte, E. & Verschaffel, L. (1985). An empirical validation of computer models of children's word problem solving (Report No. SE-045-472). Chicago, Illinois: American Educational Research Association. (69th; (ERIC Document Reproduction Service No. 255 378)

Fischbein, E. & Others (1985). The Role of implicit models in solving verbal problems in multiplication and division. Journal for Research in Mathematics Education, 16(1), 3-17.

Fitzgerald, A. & Cullinan, B. (1984). IRA Presidents take stand on misuse of readability formulas. Language Arts, 61, 883-884.

Gilbert, K. & Leitz, S. (1982). Problem solving styles among children working in small groups on mathematical word problems (Report No. SE-037-068). Stanford, California, Stanford University: California Center for Educational Research. (ERIC Document Reproduction Service No. ED 215 877)

Harris, S. (1986, July 25). Blaming technology. South Bend Tribune, pp

Herrmann, B. A. (1986). Characteristics of verbal interaction patterns of teachers of mathematical story problems. Columbia: University of South Carolina, College of Education. Paper presented to the American Education Research Association.

Herrmann, B. A. (1986). Strategic problem-solving of mathematical story problems: A descriptive study of the effects and characteristics of direct teacher explanation. Columbia: University of South Carolina, College of Education.

Johnson, D. W., Skon, L. & Johnson, R. (1980). Effects of cooperative, competitive and individualistic conditions on children's problem-solving performance. American Educational Research Journal, 17(1), 83-93.

Jones, E. D. & Others (1985). A comparison of concurrent and sequential instruction of four types of verbal math problems. Remedial and Special Education, 6.

Moser, J. M. & Carpenter, T. P. (1982). Invented processes in solution to arithmetical problems (Report No. SE-037-262). Madison, Wisconsin: Wisconsin Center for Education Research. (ERIC Document Reproduction Service No. ED 182 465)

Moser, J. M. & Carpenter, T. P. (1982). Using the microcomputer to teach problem-solving skills: Program development and initial pilot study. Report from the project on studies in mathematics. Working paper No. 328 (Report No. SE-039-730). Madison, Wisconsin: Center for Education Research. (ERIC Document Reproduction Service No. ED 224 691)

Muth, D. K. (1982). Cognitive demands that arithmetic word problems impose on children (Report No. 039-498). Marietta, Georgia: Kennesaw College, Division of Education. (ERIC Document Reproduction Service No. ED 223 422)

Nesher, P, Greeno, J. G. & Riley, M. S. (1982). The development of semantic categories for addition and subtraction. Educational Studies in Mathematics, 13(4), 373-393.

Noddings, N. (1982). The use of small group protocols in analysis of children's arithmetical problem solving (Report No. SE-037-067). Stanford, California: Stanford University, California. (ERIC Document Reproduction Service No. ED 215 876)

Papert, S. (1980). Mindstorms: Children, computers, and powerful ideas. New York: BASIC Books, Inc.

Paul, D. J. & Others (1986). The effects of adjusting readability on the difficulty of mathematics story problems. Journal for Research in Mathematics Education, 17(3), 163-171.

Quintero, A. H. (1981) Conceptual understanding in solving multiplication word problems (Report No. SE-034-831). Los Angeles, CA: Paper presented at the Annual Meeting of the American Educational Research Association, April, 1981. (ERIC Document Reproduction Service No. ED 201 497)

Quintero, A. H. (1983). Conceptual understanding in solving two-step word problems with a ratio. Journal for Research in Mathematics Education, 14(2), 102-112.

Quintero, A. H. (1984). Children's difficulties with two-step word problems (Report No. SE-044-295). New Orleans, LA: American Educational Research Association. (ERIC Document Reproduction Service No. ED 242 535)

Riley, M. S., Greeno, J. A. & Heller, J. I. (1983). Development of children's problem-solving ability in arithmetic. In H. Ginsburg (Ed.), The development of mathematical thinking (pp 153-200). New York: Academic Press.

Riley, M. S. & Greeno, J. G. (1983). Development of children's problem solving ability in arithmetic. In H. P. Ginsburg (Ed.), Development of mathematical thinking. New York: Academic Press.

Romberg, T. A. & Collins, K. F. (1985). Cognitive functioning and performance on addition and subtraction word problems. Journal for Research in Mathematics Education, 16(5), 375-382.

Schwartz, J. L./Quintero, A. H. (1981). The role of semantic understanding in solving multiplication word problems (Report No. SE-035-777). Cambridge, Massachusetts: Massachusetts Institute of Technology. (ERIC Document Reproduction Service No. 209 080)

Shalin, V. L. & Bee, N. V. (1985). Structural differences between two-step word problems (Report No. SE-045-930). Pittsburgh, PA: Pittsburgh University Learning Research and Development Center. (ERIC Document Reproduction Service No. ED 259 949)

Threadgill-Sowder, J. & Sowder, L. (1982). Drawn versus verbal formats for mathematical story problems. Journal for Research in Mathematics Education, 13(5), 324-331.

Threadgill-Sowder, J. (1983). Question placement in mathematical word problems. School Science and Mathematics, 83(2).

Threadgill-Sowder, J., Sowder, L., Moyer, J. C. & Moyer, M. B. (1985). Cognitive variables and performance on mathematical story problems. Journal of Experimental Education, 54, 56-62.

Vaidya, S. & Others (1981). Field dependence-independence and mathematics problem-solving (Report No. SE-036-083). Los Angeles, CA: American Psychological Association (89th). (ERIC Document Reproduction Service No. ED 212 473)

Wheeler, L. J. & McNutt, G. (1983). The effect of syntax on low-achieving student's abilities to solve mathematical word problems. The Journal of Special Education, 17(3), 309-315.

Wilson, F. C. (1982). The effect of a model designed to facilitate mathematical story problem skills in children in the intermediate elementary grades (Report No. SE-039-612). Los Angeles, CA: Walden University. (ERIC Document Reproduction Service No. ED 223 446)

Yancy, A. V. (1981). Pupil generated diagrams as a strategy for solving word problems in elementary mathematics (Report No. SE-045-962). Louisville, Kentucky: University of Louisville. (ERIC Document Reproduction Service No. ED 260 922)

Addendum to References

DeCorte, Erik and Lieven Verschaffel (1986). Eye - movement data as access to solution processes of elementary addition and subtraction problems (SE 046 906). San Francisco, California, U.S.A.: American Educational Research Association. (ERIC Document Reproduction Service No. ED 273 450)

DeCorte, Erik and Lieven Verschaffel (1987). The influence of some non-semantic factors on solving addition and subtraction word problems (SE 047 953). Washington, D.C. U.S.A.: American Educational Research Association. (ERIC Document Reproduction Service No. ED 281 759)

Fennema, Elizabeth and Lindsay A. Tartre (1985). The use of spatial visualization in mathematics by girls and boys. Journal for Research in Mathematics Education 16 (3), 184-206.

Lean, G.A. and Clements, M.A. (1981). Spatial ability, visual imagery, and mathematical performance (Report No. 10). Lae: Papua New Guinea University of Technology, Mathematics Education Centre.

Muth, Denise K. (1986). Solving word problems: middle school students and extraneous information. School Science and Mathematics 86 (2), 108-111.

Paul, Douglas J. (1986). The effects of adjusting readability on the Difficulty of mathematics story problems. Journal for Research in Mathematics Education, 17 (3), 163-171.

Rathnell, Edward C. (1981). Teaching children to solve one-step word problems (SE 035 726). Cedar Falls, Iowa, U.S.A.: The University of Northern Iowa. ERIC Document Reproduction Service No. ED 207 849

van Lieshout, Ernest. (1986). Developing a computer assisted strategy training procedure for children with learning deficiencies to solve addition and subtraction word problems (SE 046 623). San Francisco, California, U.S.A.: American Educational Research Association. (ERIC Document Reproduction Service No. ED 271 285)

Wright, Jone P. and C. Dan Wright (1986). Personalized verbal problems: an application of the language experience approach. Journal of Educational Research, 79 (6), 358-362.

Zollman, Alan (1987). Aspects of transfer of learning in mathematical problem solving with respect to the order of problem presentation (SE 047 942). Washington, D.C., U.S.A., American Educational Research Association. (ERIC Document Reproduction Service No. ED 281 748).

A P P E N D I X

THE THREE GENERAL SEMANTIC CATEGORIES OF ADDITION AND SUBTRACTION WORD PROBLEMS*

Name of The Category	Characteristics	Example
1. Combine	Involves a Static relationship between sets. Asking about the Union set or about one of two disjoint sub-sets.	There are 3 boys and 4 girls. How many children there are altogether?
2. Change	describes increase or decrease in some initial state to produce a final state.	John has 7 marbles. He lost 3 of them. How many marbles does John have now?
3. Compare	involves a static comparison between two sets. Asking about the difference set or about one of the sets where the difference set is given.	Tom has 6 marbles. Joe has 4 marbles. How many marbles does Tom have more than Joe?

*Nesher, P. and Others (1982).